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EXPANDING AREA SEARCH EXPERIMENTS

by

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The formula $1 - \exp(-(VW/\pi U^2)(1/\tau - 1/t))$ is often used to approximate the probability of detecting a target with speed U by time t if the search does not start until time τ and the searcher's speed and sweep width are V and W, respectively. This report shows some experimental evidence that the formula is an imperfect but reasonably good approximation to what actually happens when the target is evasive.

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EXPANDING AREA SEARCH EXPERIMENTS

by

Alan R. Washburn

Introduction.

With fast computers and new algorithms, it has recently become possible to optimize the distribution of effort when searching for a moving target, particularly if the target's motion is Markov [1,5,6]. There has also been some recent work on optimizing the search path (rather than the distribution of effort), but this problem seems to be inherently more difficult [3,4]. No general approach to the problem of searching for a target whose motion is worst case currently exists, either in the distribution of effort case or in the search path case. The lack of progress on the worst case problem should not be surprising in view of the general intractability of two person zero sum games; it is nonetheless unfortunate because many applications of search theory are to problems where the motivation of the target makes the worst case formulation natural.

In spite of the lack of general methods, certain specific two person zero sum search games have been solved or approximated. One of those that has been approximated is the problem of searching in an expanding area, which is the subject of this report. An Evader knows that he has been spotted at time 0, and proceeds to maneuver at speed U in order to evade the subsequent effort to (re)-detect him. The Pursuer must wait until time τ before beginning to search, after which he searches until time t at speed V and sweep width W (he has to come within $W/2$ of the target) in an effort to detect the target. The classic example is the "flaming datum" problem in antisubmarine warfare. Coggins [2] gives the formula

$$(1) \qquad p_d = 1 - \exp(-(VW/\pi U^2)(1/\tau - 1/t))$$

for the probability of detection in such a search. Briefly, (1) can be derived by reasoning that $(VWdv)/(\pi U^2 v^2)$ is the ratio of (area searched in dv) to (area of farthest-on circle) at time v, and is therefore the probability of detection in time dv. The average number of detections in the interval $[0,t]$ is therefore

$$n(t) \equiv \int_\tau^t (VW\,dv)/(\pi U^2 v^2) = (VW/\pi U^2)(1/\tau - 1/t) \quad \text{for} \quad t \geq \tau .$$

Assuming that detections in non-overlapping intervals are independent, the number of detections in $[0,t]$ is a Poisson random variable, and the probability of no detections in $[0,t]$ is therefore $\exp(-n(t))$. Formula (1) is then the probability that the number of detections in $[0,t]$ is not 0. The main point of the above sketch of Coggins' derivation is that some assumptions are required to derive (1), one of which (independence in non-overlapping intervals) is questionable if searcher and target must each have a continuous path.

Furthermore, the derivation offers no clue to optimal tactics for either target or searcher, except perhaps that the searcher should search "randomly" so that the crucial independence assumption is satisfied.

Coggins derived (1) using the assumption that a random search was employed, in which case the type of motion used by the Evader is immaterial. Similarly, if the Evader could move in such a manner that his position was uniformly distributed over the farthest-on circle at all times and independent at closely spaced times, then (1) would hold regardless of the Pursuer motion; that is, (1) would be a saddle point if random search and random Evader motion of that type were feasible. Such strategies are not feasible. Nonetheless, given the typical insensitivity of payoff to strategy choice in the vicinity of a saddle point, (1) is at least somewhat plausible as an approximation to the value of the game.

Given the facts that (1) is commonly used and that its derivation is plausible but questionable, some validation effort seems warranted. An attempt to do this has been carried out at NPS over the last several years using officer-students as subjects in an electronic version of the game. The next section gives a complete description of the experiment, but a quick summary could be obtained by simply inspecting Figures 2-5, which show experimental vs theoretical (formula (1)) results for several combinations of parameters.

The Experiment

Since there are two physical quantities involved (length and time), two of the five parameters can be set to convenient constants without loss of generality. Our choice was to set $\tau = 10$ seconds and $U = .024$ units/second in all trials; the definition of the length unit is immaterial in (1), but in fact a "unit" is about 5 inches in all experiments. In 60 seconds the farthest-on circle therefore has a radius of about 7.2 inches, which fills up the screen. The parameters V and W were then varied to obtain Figures 2-5, with $0 \leq t \leq 60$ seconds in each figure. Capture time was recorded to the nearest second in Figures 2-3 and (when it was realized that greater accuracy was appropriate) to the nearest .1 second in Figures 4-5.

Figure 1 shows the experimental setup. Each subject see his own position and the constantly expanding farthest-on circle displayed on a cathode ray tube, with his velocity being controlled by joystick up to the appropriate limit. In addition, the Pursuer sees a capture circle around his own position as a visual aid if he should decide to use a spiral track that makes the capture circle tangent to the farthest-on circle. The Pursuer starts at the center of the screen, and finds his joystick "dead" for the time late τ. In a few cases, in spite of being warned about what would happen,

Figure 1

the Evader (who also starts at the center of the screen), was less than $W/2$ from the center of the screen by time τ, in which case the capture time was recorded as τ. If capture had not occurred by 60 seconds, the trial was terminated.

The subjects were officer-students enrolled in certain courses taught by the author over the period 1978-80. The differences in sample size in Figures 2-5 are due mainly to class size differences. The subjects typically played in pairs, spending half an hour in each role. The only instruction given to the subjects was to caution them about the obvious mistakes of not initially leaving the center on the part of the Evader or of searching outside the farthest-on circle on the part of the Pursuer; the idea was to determine what happens when tactics are whatever comes naturally. In spite of the simple nature of the game, some learning about tactics did take place. For example, one subject Pursuer made fruitless spiral sweeps several trials in a row before realizing that the Evader had stumbled on the strategy of returning to the center and staying there, which was of course reinforced when no detection occurred. This subject then realized that unpredictability is an important part of tactics and quit searching for the "optimal" track. While this sort of learning is of course good from a tutorial point of view, the reader should realize that Figures 2-5 include data from a wide variety of subjects, some "skilled" and some "unskilled."

The following points seem to characterize good Pursuer tactics in this game:

1) Stay inside the farthest-on circle. Any area covered outside is wasted.
2) Always go at top speed. The Evader is blind in any case, so there is no advantage to going slow.
3) Keep the radius of curvature large. The danger that a slow, blind Evader will slip into a recently covered area is small compared to the danger of wasting effort due to redundant coverage.

A Pursuer that followed 2) and 3) exactly would go in a straight line and therefore not follow 1). The three points are therefore in conflict, and resolution of the conflict is the art of playing the game for the Pursuer. Playing the Pursuer part is not trivial; an experiment with $V = .384$ unit/second had to be rejected because most players could not follow 1) and 2) simultaneously in the vital seconds at the beginning of the game.

The following points seem to characterize good Evader tactics

1) Initially, pick a direction at random and go at top speed for a while. Take advantage of the time late.
2) Sometimes, stay on the farthest-on circle throughout the game. At other times pause occasionally in the process of fleeing.

It is vital that the Evader follow point 1), and important that the Evader not always be found on the farthest-on circle, since there is sufficient time in all cases for the Pursuer to sweep it. Otherwise, it does not seem to matter a whole lot what the Evader does. After the first few seconds, there are not many mistakes an Evader can make.

The fact that the game seems to punish unskilled Pursuers more than unskilled Evaders may explain why the experimental curve lies so far below theoretical in Figure 5. A Kolmogorov-Smirnov test at the .05 level would reject that curve but not the curves in Figures 2, 3, or 4. With this slim guidance, we leave the reader to come to his own conclusions about the validity of (1). Our own intention is to use it until something better comes along, at least in situations where the Evader knows when the search begins.

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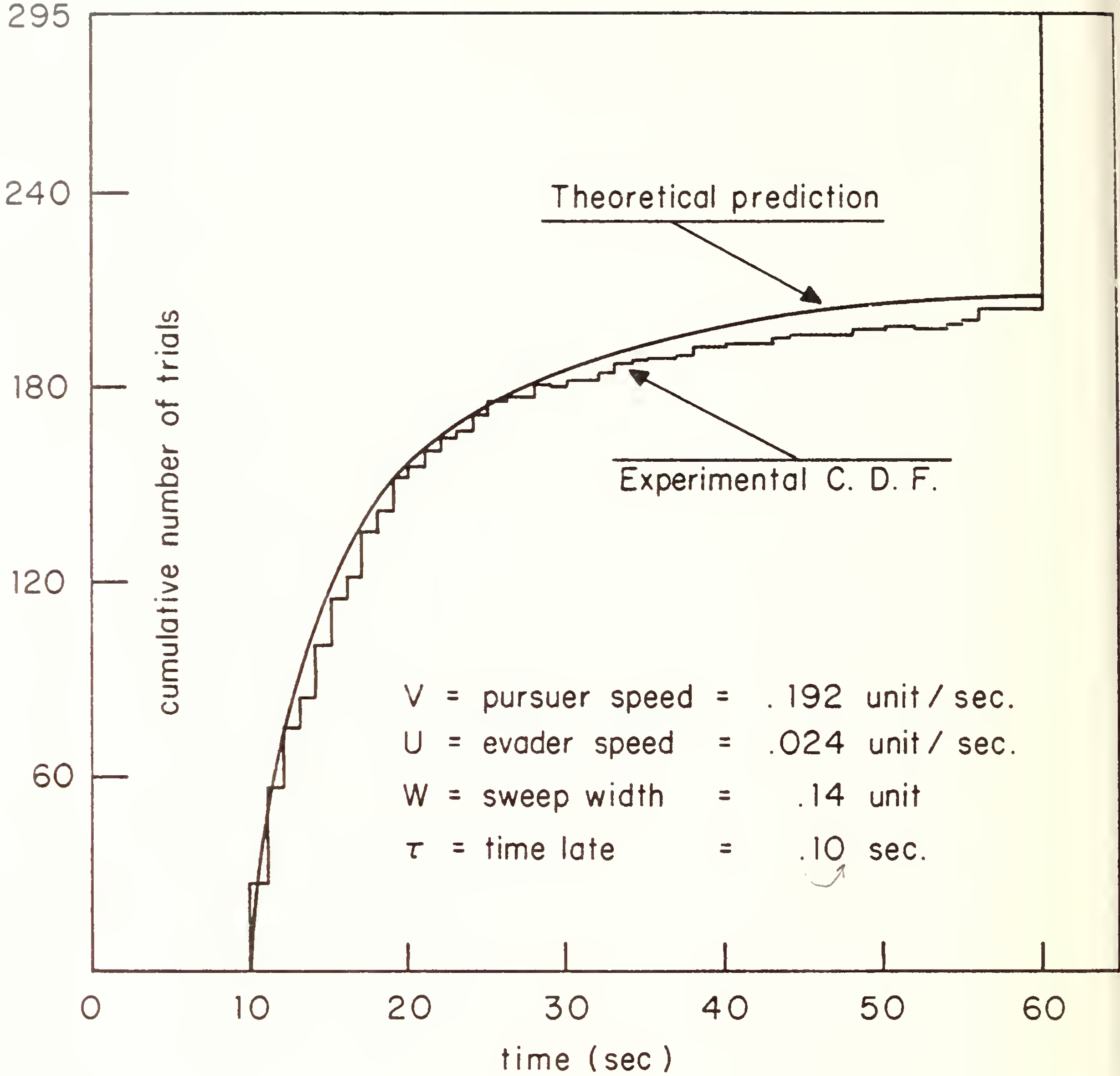
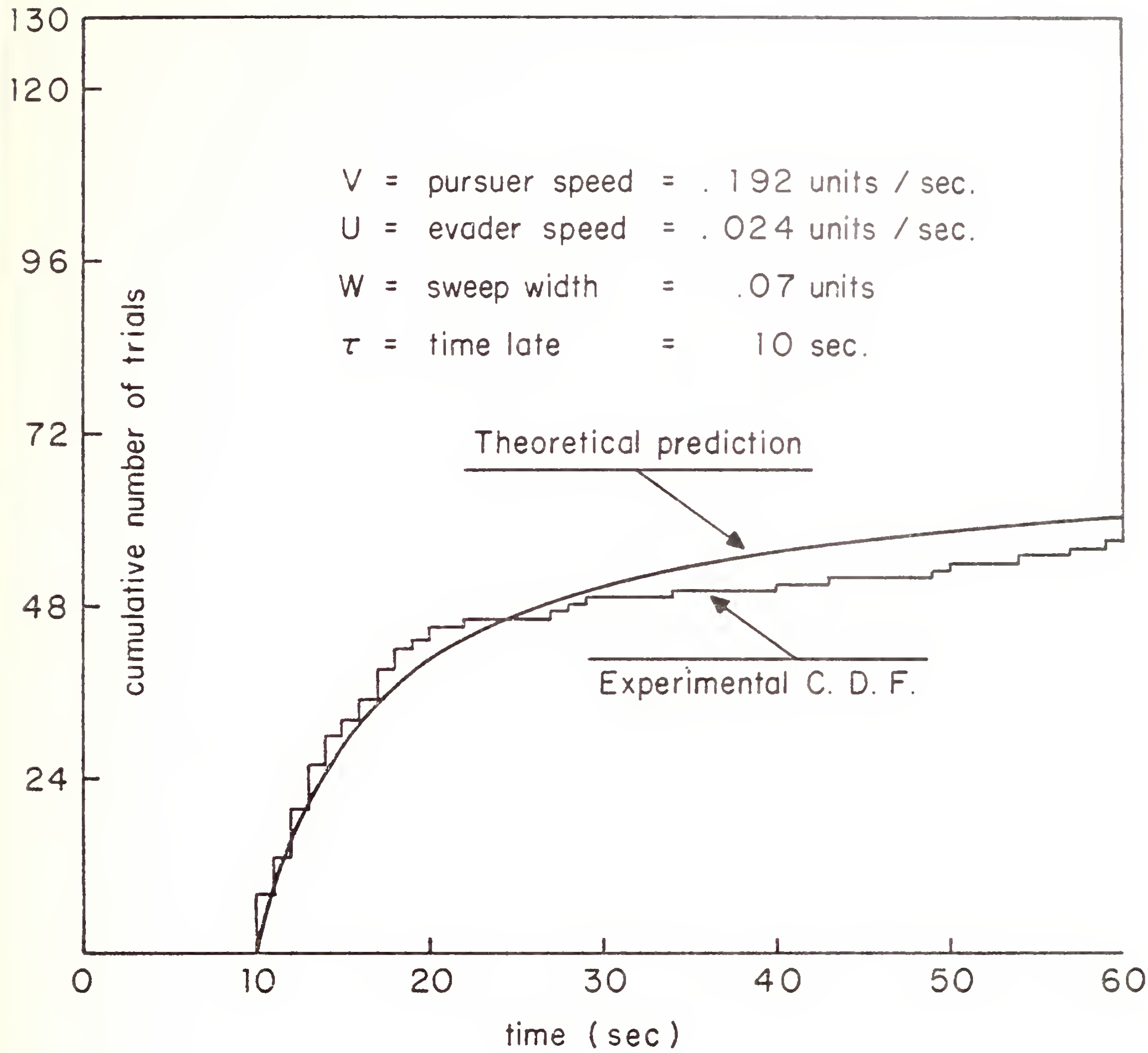
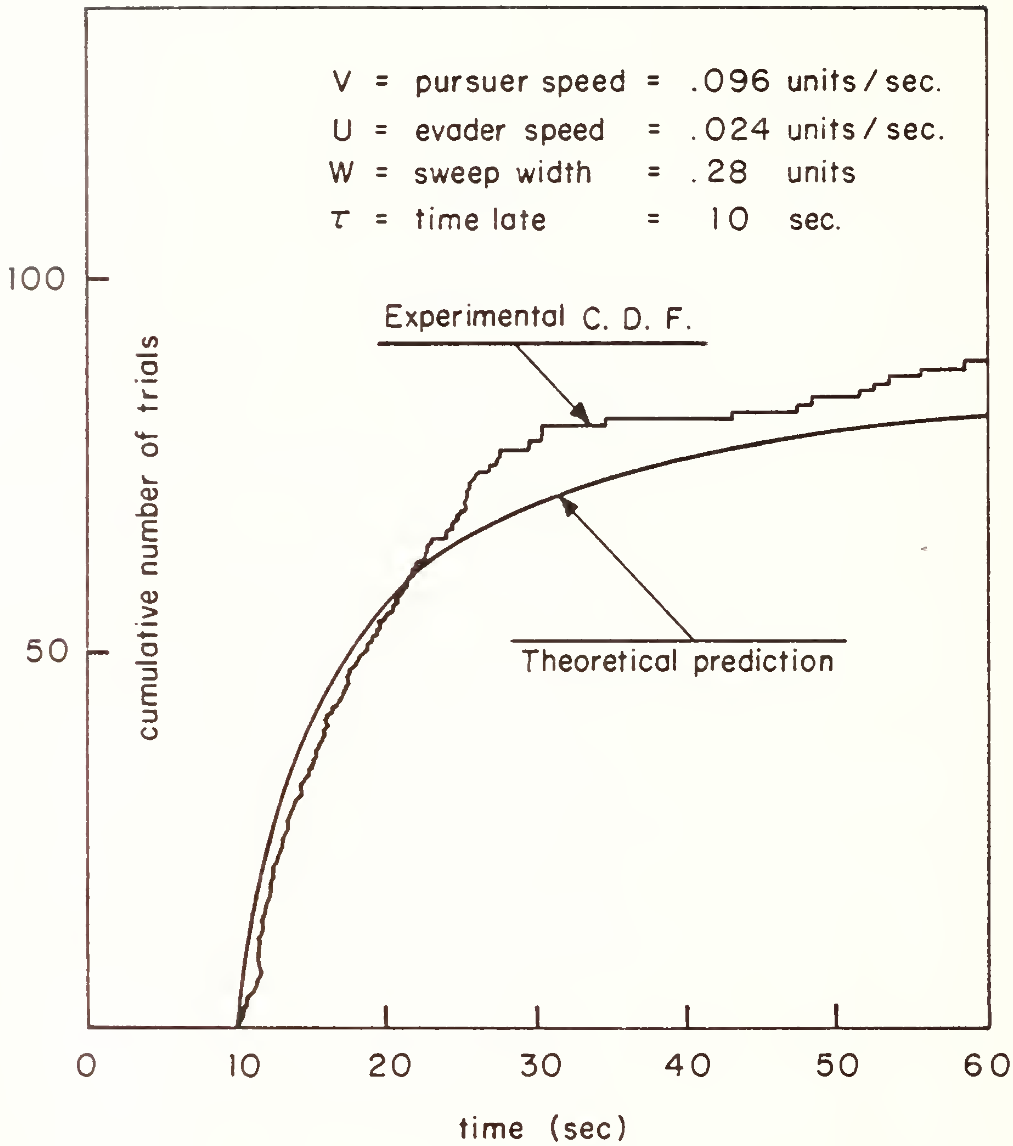

295
240
180
120
60
cumulative number of trials
Theoretical prediction
Experimental C. D. F.
V = pursuer speed = .192 unit / sec.
U = evader speed = .024 unit / sec.
W = sweep width = .14 unit
τ = time late = .10 sec.
0
10
20
30
40
50
60
time (sec)

Figure 2.



130
120
96
72
48
24
0
10
20
30
40
50
60
cumulative number of trials
time (sec)
V = pursuer speed = .192 units / sec.
U = evader speed = .024 units / sec.
W = sweep width = .07 units
τ = time late = 10 sec.
Theoretical prediction
Experimental C. D. F.

Figure 3.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U = evader speed = .024 units / sec.
W = sweep width = .28 units
τ = time late = 10 sec.
Experimental C. D. F.
Theoretical prediction
cumulative number of trials
100
50
0
10
20
30
40
50
60
time (sec)

Figure 4.

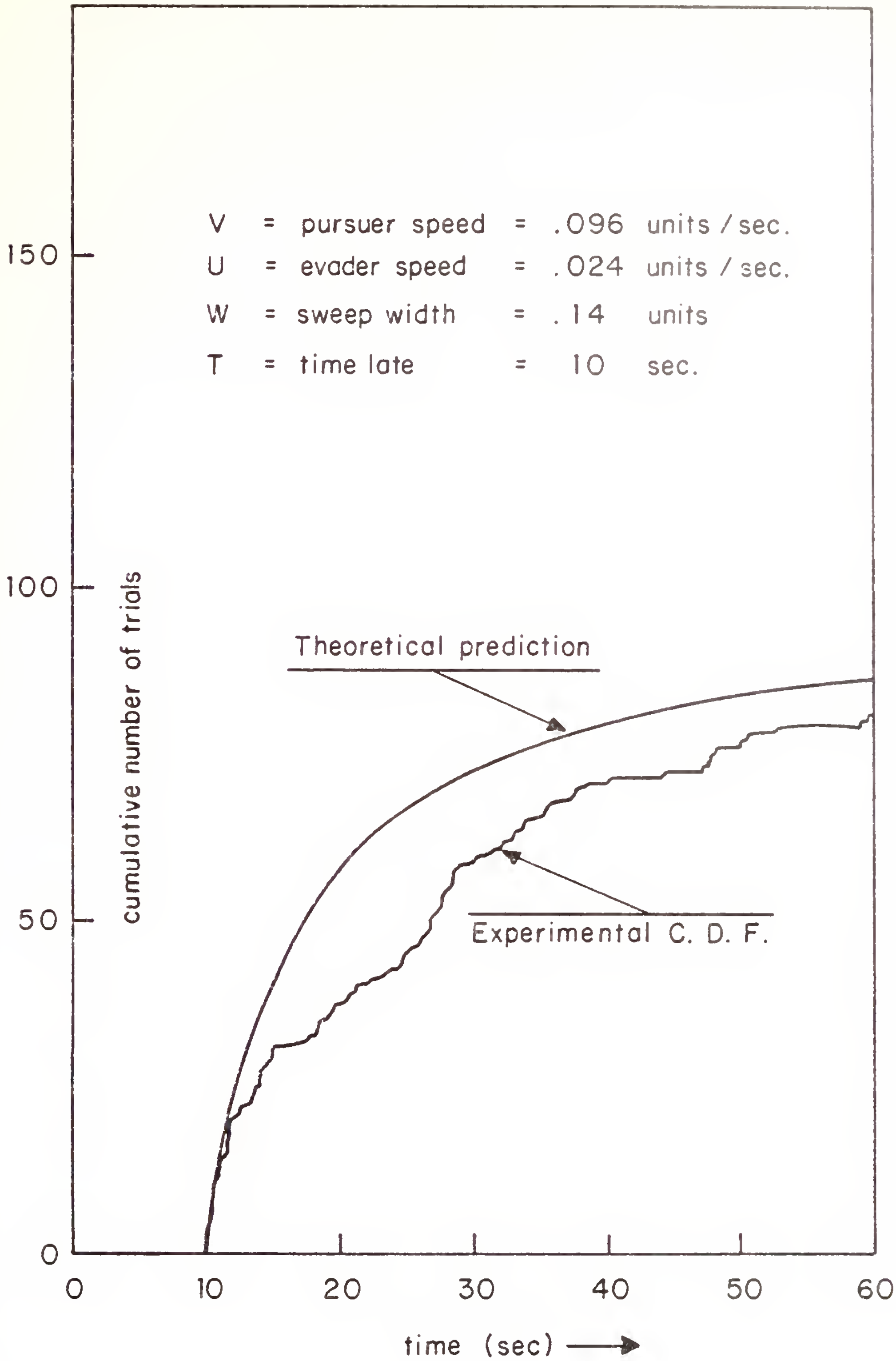
V = pursuer speed = .096 units / sec.
U = evader speed = .024 units / sec.
W = sweep width = .14 units
T = time late = 10 sec.
150
100
50
0
cumulative number of trials
Theoretical prediction
Experimental C. D. F.
0
10
20
30
40
50
60
time (sec)

Figure 5.

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